

Can ‘Intrinsic’ Be Defined Using Only Broadly Logical Notions?¹

DAN MARSHALL
July 27, 2007

Abstract

An intrinsic property is roughly a property things have in virtue of how they are, as opposed to how they are related to things outside of them. This paper argues that it is not possible to give a definition of ‘intrinsic’ that involves only logical, modal and mereological notions, and does not depend on any special assumptions about either properties or possible worlds.

1 Introduction

A thing may have a property in virtue of how it is, or in virtue of how things are outside of it, or in virtue of how it is related to things outside of it. Consider, for example, a piece of tin that is next to a piece of gold. The piece of tin has the property Is made of tin in virtue of how it is; whereas, it has the property Is such that there is a piece of gold in virtue of how things are outside of it, and it has the property Is next to something in virtue of how it is related to things outside of it. A thing can either partly or wholly have a property in one of these ways. For example, a thing may have a property *wholly* in virtue of how it is, or it may have it merely *partly* in virtue of how it is. The piece of tin has Is made of tin wholly in virtue of how it is; whereas, it has the property Is made of tin and next to something merely partly in virtue of how it is. The piece of tin has Is made of tin wholly in virtue of how it is since how it is determines that it has the property. In contrast, the piece of tin has Is made of tin and next to something merely partly in virtue of how it is since how the tin is contributes to the tin having this property without determining that it has the property: in addition to the tin being how it is, the tin must be next to something in order to have the relevant property. A thing may also have a property in more than one of these ways. For example, the piece of tin has the property Is made of tin and next to something both partly in virtue of how it is and partly in virtue of how it is related to other things, and it has the property Is either made of tin or next to something both

¹This paper presents an argument that ‘intrinsic’ cannot be defined using only broadly logical notions. Josh Parsons has given a similar argument in (Parsons 2001, p. 22-23). Although Parson’s argument is briefer and more informal than the argument presented here, it involves essentially the same underlying idea. Thanks are due to Martin Davies, Josh Parsons, Peter Roeper and Daniel Stoljar for their valuable comments.

wholly in virtue of how it is and wholly in virtue of how it is related to things outside of it.

A thing has a property *intrinsically* iff it has the property wholly in virtue of how it is. A thing has a property *extrinsically* iff it has the property non-intrinsically: that is, iff it has the property without having it wholly in virtue of how it is. For example, the piece of tin has Is made of tin and Is either made of tin or next to something intrinsically; whereas, it has the properties Is next to something, Is such that there is a piece of gold and Is made of tin and next to something extrinsically. A property is *intrinsic* iff, necessarily, anything that has the property has it intrinsically. A property is *extrinsic* iff it is not intrinsic: that is, iff it is possible for something to have the property without having it wholly in virtue of how it is. The property Is made of tin is intrinsic, whereas the properties Is either made of tin or next to something, Is next to something, Is such that there is a piece of gold and Is made of tin and next to something are all extrinsic.

It would be good if we could define ‘intrinsic’ using notions that are better understood, and less closely related to the notion of an intrinsic property, than notions like ‘a thing having a property wholly in virtue of how it is’. This could be achieved if we could define ‘intrinsic’ using only *broadly logical* notions, where broadly logical notions are those notions that can be expressed using the following vocabulary: the logical vocabulary of first order predicate logic; the predicates ‘is a possible world’, ‘is a set’, ‘exists’, ‘=’, ‘∈’, ‘is a proper part of’, ‘instantiates’ and ‘is a property’; and the modal operators ‘possibly’, ‘necessarily’ and ‘at’.² This paper will discuss whether this can be done.

It is possible to define ‘intrinsic’ using only broadly logical notions given special assumptions about properties. (Or, at least, it is possible to give an account of the form ‘ p is intrinsic iff $\phi(p)$ ’ that is necessary and where ϕ contains only broadly logical vocabulary, given special assumptions about properties.³) For example, if it is necessary that

² A more precise statement of what counts as a broadly logical definition of ‘intrinsic’ is the following. Let L be a language containing the variables ‘ x ’, ‘ y ’, ‘ z ’, ‘ x_1 ’... together with the operators and predicates: ‘ \neg ’ (meaning ‘it is not the case that’), ‘ \wedge ’ (meaning ‘and’), ‘ \exists ’ (meaning ‘for some’), ‘ \Box ’ (meaning ‘necessarily’), ‘at’, ‘is a possible world’, ‘is a set’, ‘exists’, ‘=’, ‘ \in ’ (meaning ‘is a member of’), ‘is a proper part of’, ‘instantiates’, and ‘is a property’. If X and Y are variables in L then ‘ $(X$ is a possible world)’, ‘ $(X$ is a set)’, ‘ $(X$ exists)’, ‘ $(X = Y)$ ’, ‘ $(X \in Y)$ ’, ‘ $(X$ is a proper part of $Y)$ ’, ‘ $(X$ instantiates $Y)$ ’, and ‘ $(X$ is a property)’ are atomic formulas in L . If A and B are formulas in L , and X is a variable in L , then ‘ $\neg A$ ’, ‘ $(A \wedge B)$ ’, ‘ $(\exists X)A$ ’, ‘ $\Box A$ ’, and ‘(at X) A ’ are formulas in L . All formulas in L are specified by the previous two sentences. Note that I do not need to include operators like ‘ \vee ’ (meaning ‘or’), ‘ \forall ’ (meaning ‘for all’), or ‘ \Diamond ’ (meaning ‘possibly’) since they can be defined in terms of the primitive vocabulary of L . I can now say that ‘intrinsic’ can be defined using only broadly logical notions iff ‘ x is intrinsic’ is an abbreviation of a formula in L . We might wish to broaden what counts as a broadly logical definition either by i) turning L into an infinitary language by allowing infinite conjunctions and infinite blocks of quantifiers, or ii) by adding vocabulary such as the plural quantifier ‘there are’, the operator ‘actually’, or the term ‘the actual world’ to the list of broadly logical vocabulary. The argument of this paper can presumably be modified so that it applies to various such expanded conceptions of what counts as a broadly logical definition.

³Whether such an account counts as a definition depends on what more, if anything, is needed for a

there are no extrinsic properties—so that predicates like ‘is next to a tin’ fail to express properties—then we can give the following account: all properties are intrinsic. Similarly, if it is necessary that extrinsic properties have proper parts while intrinsic properties do not have proper parts, then we can say that a property is intrinsic iff it has no proper parts.⁴ It might also be possible to define ‘intrinsic’ given special assumptions about possible worlds.⁵

While it is possible to define ‘intrinsic’ using only broadly logical notions given special (and highly controversial) assumptions about properties, and it might be possible to define ‘intrinsic’ using only broadly logical notions given similarly special (and controversial) assumptions about possible worlds, it would be desirable to define ‘intrinsic’ without making such assumptions. Jaegwon Kim (1982) has proposed an account of ‘intrinsic’ that, if successful, would provide such a definition. We have the following definitions. Something is *contingent* iff it contingently exists: that is, iff it exists but may not have existed. Two things are *wholly distinct from one another* iff they have no parts in common. Something is *accompanied* iff it coexists with a contingent thing wholly distinct from itself. Finally, something is *lonely* iff it does not coexist with any contingent thing wholly distinct from itself.⁶ Kim’s suggestion, in effect, is that a property is intrinsic iff it is possible for something to have the property and be lonely.⁷ Kim’s proposal involves only broadly logical notions since it can be written as: F is intrinsic iff Possibly $[\exists x[[x$ instantiates $F] \wedge \neg \exists y[\neg(y = x) \wedge \neg \exists z((z \text{ is a proper part of } x) \wedge (z \text{ is proper part of } y))]]]$.

Unfortunately, Kim’s definition does not work. As David Lewis (1983a) has pointed out, the property Is lonely is extrinsic—it is a property a thing has, not in virtue of how it is, but in virtue of how things are outside of it. But clearly it is possible for something to have the property Is lonely and be lonely.⁸ So Kim’s definition falsely classifies Is lonely

necessarily true account to be a definition. I will not address this issue here.

⁴Cf (Sider 1996, n. 31).

⁵Stephen Yablo (1999) has proposed a definition of ‘intrinsic’ using only broadly logical notions that depends on a genuine modal realist theory of possible worlds as concrete entities. The genuine modal realist view required by Yablo’s definition differs from Lewis’s well known genuine realist theory of possible worlds (Lewis 1986) by allowing overlap between worlds. Yablo claims to be able to formulate his definition so that it does not presuppose any such special assumptions about worlds. However, his reformulated definition does not count as a broadly logical definition of ‘intrinsic’ according to the classification used here. For further discussion of Yablo’s account of ‘intrinsic’ see (Marshall ms).

⁶The terms ‘lonely’ and ‘accompanied’ are due to Lewis (1983a).

⁷I follow Lewis (1983a) in simplifying Kim’s formulation by assuming a temporal parts theory of persistence, according to which things that persist through time consist of wholly distinct temporal parts at different times. According to Kim’s original formulation: a property F is *rooted outside times at which it is had* iff, necessarily, for any object x , for any time t , x has the property F at t only if x exists at some time before or after t ; F is *rooted outside the objects that have it* iff, necessarily, for any object x , x has F only if some contingent object wholly distinct from x exists; F is intrinsic—Kim uses the term ‘internal’—iff F is neither rooted outside times at which it is had nor rooted outside the objects that have it.

⁸Given certain metaphysical views, it is impossible for something to be lonely, and hence impossible for something to have Is lonely and be lonely. For example, it will be impossible for something to be lonely if it is necessary that: i) there are contingent things, ii) for any x , if x is contingent then $\{x\}$ is contingent,

as intrinsic.⁹

Kim's proposed definition fails. However, there might be some other way of defining 'intrinsic' using only broadly logical notions that does not depend on any special assumptions about properties or possible worlds. I will argue that, unfortunately, this is not the case. I will do this by arguing that, given widely held views about these entities, it is not possible to define 'intrinsic' using only broadly logical notions.

It will be convenient to make some specific assumptions about properties, relations, possible worlds and ontology. First, I will assume that possible worlds (or worlds, for short) are maximal possible abstract states of affairs, and that 'possibly' and 'necessarily' can be defined in the standard way in terms of quantification over worlds.¹⁰ Second, I will assume the following familiar theory of properties and relations. An n -ary relation is a set of $n + 1$ -tuples $\langle x_1, \dots, x_n, w \rangle$, where w is a world. A property is a 1-ary relation. An n -tuple $\langle x_1, \dots, x_n \rangle$ instantiates an n -ary relation R iff $\langle x_1, \dots, x_n \rangle$ instantiates R at the actual world. An n -tuple $\langle x_1, \dots, x_n \rangle$ instantiates an n -ary relation R at a world w iff $\langle x_1, \dots, x_n, w \rangle \in R$. All instances of the following schema are true:

Suppose ' ϕ ' is an n -place predicate. Then ' ϕ ' expresses a relation R iff, for any world w , and for any x_1, \dots, x_n , (at w , $\phi x_1 \dots x_n$) iff ($\langle x_1, \dots, x_n \rangle$ instantiates R at w).

I adopt the convention that an underlined predicate is a name for the relation expressed by the predicate. So, for example,

$$\underline{\text{Is an electron}} = \{ \langle x, w \rangle \mid \text{at } w, x \text{ is an electron} \}.$$

Third, I will assume that there is the standard set-theoretic hierarchy of sets, these being constructed iteratively out of the empty set and singleton sets of concrete things and worlds. Fourth, I will assume that i) necessarily, everything is either a concrete thing, a world or a set, and that ii) necessarily, nothing is more than one of these things (that is,

and iii) every set is wholly distinct from its members. If it is impossible for something to be lonely then Kim's definition will still fail, since it will falsely classify all properties as extrinsic.

⁹Lewis also showed that the most obvious ways of trying to fix Kim's definition also fail.

¹⁰ S is a *maximal* state of affairs iff, for every state of affairs S^* , either S entails S^* or S is incompatible with S^* ; S entails S^* iff, necessarily, if S obtains then S^* obtains; S is *incompatible with* S^* iff, necessarily, if S obtains then S^* does not obtain; S is *possible* iff it is possible for S to obtain. The theory of worlds assumed here is similar to that of Alvin Plantinga (1974). It is assumed that, necessarily, only one world obtains, and that the unique world that obtains is the actual world. The operators 'necessarily' and 'possibly' can be defined in terms of quantification over worlds and the 'at' operator as follows: (Possibly, A) iff, for some world w , at w , A ; (Necessarily, A) iff, for any world w , at w , A . The 'at' operator can in turn be defined in terms of 'necessarily' as follows: (at x , A) iff x is a world and, necessarily, if x obtains then A . Note that it follows from this definition of 'at' that \lceil at x , A \rceil is false if x is not a world. So, according to this definition, sentences like 'At the Eiffel Tower, grass is green' are false.

necessarily, nothing is both a concrete thing and a world, nothing is both a concrete thing and a set, and nothing is both a world and a set).¹¹

The choice of these assumptions is largely inessential to the argument presented here. The argument can be made to work with a wide variety of alternative views about properties, relations, worlds and ontology. For example, the argument can be made to work with theories of properties according to which properties are basic entities distinct from sets,

¹¹ In addition to these assumptions, and the assumptions stated in footnote 7 and footnote 10, I also make the following assumptions.

(1) I assume a possibilist theory of what there is according to which there are things that might have existed but do not actually exist. Without this assumption, the familiar theory of properties and relations described in the main text is implausible.

(2) I assume that the formulas in language L defined in footnote 2 have the following recursive truth conditions. For any assignment a to the variables in L , for any variables X and Y in L , for any formulas A and B in L , and for any world w : i) $\ulcorner(X \text{ is a possible world})\urcorner$ is $\text{true}_{a,w}$ iff $a(X)$ is a possible world; ii) $\ulcorner(X \text{ exists})\urcorner$ is $\text{true}_{a,w}$ iff $a(X)$ exists at w ; iii) $\ulcorner(X = Y)\urcorner$ is $\text{true}_{a,w}$ iff $a(X) = a(Y)$; iv) $\ulcorner(X \text{ is a proper part of } Y)\urcorner$ is $\text{true}_{a,w}$ iff $a(X)$ is a proper part of $a(Y)$ at w ; v) $\ulcorner(X \text{ is a set})\urcorner$ is $\text{true}_{a,w}$ iff $a(X)$ is a set; vi) $\ulcorner(X \in Y)\urcorner$ is $\text{true}_{a,w}$ iff $a(X) \in a(Y)$; vii) $\ulcorner(X \text{ is a property})\urcorner$ is $\text{true}_{a,w}$ iff $a(X)$ is a property; viii) $\ulcorner(X \text{ instantiates } Y)\urcorner$ is $\text{true}_{a,w}$ iff $a(Y)$ is a property and $\langle a(X), w \rangle \in a(Y)$; ix) $\ulcorner\neg A\urcorner$ is $\text{true}_{a,w}$ iff A is not $\text{true}_{a,w}$; x) $\ulcorner(A \wedge B)\urcorner$ is $\text{true}_{a,w}$ iff A is $\text{true}_{a,w}$ and B is $\text{true}_{a,w}$; xi) $\ulcorner\exists X A\urcorner$ is $\text{true}_{a,w}$ iff, for some assignment b to the variables in L differing from a at most in its assignment to x , A is $\text{true}_{b,w}$; xii) $\ulcorner\Box A\urcorner$ is $\text{true}_{a,w}$ iff for any world w' , A is $\text{true}_{a,w'}$; xiii) $\ulcorner\text{at } X, A\urcorner$ is $\text{true}_{a,w}$ iff A is $\text{true}_{a,a(X)}$; xiv) A is true_a iff A is $\text{true}_{a,\text{the actual world}}$; xv) A is true iff A does not contain any free variables and A is true_a .

Note that this account gives the truth conditions of the sentences in L without using the modal operators ‘Necessarily’ and ‘at’. Instead, the account uses the definite description ‘the actual world’, and the predicates ‘exists at’ and ‘is a proper part of ... at’. I assume that ‘exists at’ and ‘is a proper part of ... at’ are extensional: that is, they obey the inference rules of classical predicate logic with identity. I also assume that the truth conditions of any extension of L obtained by adding extensional predicates to L is given by adding to the above account extra clauses of the form:

$$\ulcorner\phi(X_1, \dots, X_n)\urcorner \text{ is } \text{true}_{a,w} \text{ iff } \phi'(a(X_1), \dots, a(X_n), w),$$

where ϕ is a n -place extensional predicate that has been added to L , and $\ulcorner\phi'(x_1, \dots, x_{n+1})\urcorner$ is a $n+1$ -place extensional formula that is not in L and that is necessarily equivalent to $\ulcorner\text{at } x_{n+1}, \phi(x_1, \dots, x_n)\urcorner$. (A is necessarily equivalent to B iff A is necessarily equivalent to B under all assignments to the variables of A and B .)

Note that it follows from the truth-conditions of L given above that, for any x and y , and for any world w : a) (at w , x is a world) iff x is a world; b) (at w , $x = y$) iff $x = y$; c) (at w , x is a set) iff x is a set; d) (at w , $x \in y$) iff $x \in y$; and e) (at w , x is a property) iff x is a property. Note, also, that it follows from a) and c), and the assumption that necessarily everything is either a concrete thing, a world or a set and nothing is more than one of these things, that for any x , and for any world w , x is concrete at w iff x is concrete.

(3) I make the following assumptions about concrete things, worlds and sets: i) For any concrete thing x , x exists at some world, and x fails to exist at some world. ii) For any world w , w exists at all worlds. iii) For any set S , S exists at all worlds. iv) For any x and y , if x is a proper part of y at some world, then x and y are concrete things at that world.

The assumptions (1-3) are chosen for simplicity and may be replaced by a wide variety of alternative views about these matters. For example, the argument can be made to work with actualist theories of what there is, actualistically acceptable theories of properties and relations, actualistically acceptable semantic theories of L , counterpart theoretic semantic theories of L , and alternative views about the existence and membership conditions of concrete things, worlds and sets.

theories of properties according to which there are distinct properties that are necessarily coextensive with each other, theories of worlds according to which worlds are consistent sets of sentences, and theories of worlds according to which worlds are maximally spatiotemporally interrelated concrete wholes. Note, though, since there are views of properties (and perhaps views of worlds) according to which ‘intrinsic’ can be defined using only broadly logical notions, the argument cannot be made to work with every theory of these entities.

I will proceed as follows: In section 2, I will give an informal version of the argument that ‘intrinsic’ cannot be defined using only broadly logical notions. In section 3, I will discuss the notion of an isomorphism with respect to a set of relations. This notion will be used in section 4 to give a more rigorous and complete version of the argument given in section 2. In section 5, I will describe how the scope of this argument can be extended so that, for example, the argument shows that ‘intrinsic’ cannot be defined using basic spatiotemporal notions as well as broadly logical notions. In section 6, I will then discuss how this type of argument can be applied more broadly to notions other than that of an intrinsic property.

2 An Informal Argument

I begin by giving an informal argument that ‘intrinsic’ cannot be defined using only broadly logical notions. For simplicity, let us suppose that there are only three concrete things a , b and c , and only six worlds $w_1, w_2 \dots w_6$. Let us also suppose the following:

1. At w_1 , a is an electron, and b and c do not exist.
2. At w_2 , a is a positron, and b and c do not exist.
3. At w_3 , b is an electron, and a and c do not exist.
4. At w_4 , b is a positron, and a and c do not exist.
5. At w_5 , a is an electron, b is a positron, and c is the composite of a and b .
6. At w_6 , a is a positron, b is an electron, and c is the composite of a and b .¹²

Consider the properties:¹³

$$\begin{aligned} \underline{\text{Is an electron}} &= \{ \langle x, w \rangle \mid x \text{ is an electron at } w \} \\ &= \{ \langle a, w_1 \rangle, \langle b, w_3 \rangle, \langle a, w_5 \rangle, \langle b, w_6 \rangle \}, \end{aligned}$$

¹²I am assuming that at these worlds i) if something is either an electron or a positron then it exists; ii) if something is either an electron or a positron then it has no proper parts; and iii) nothing is both an electron and a positron.

¹³Something is an accompanied electron iff it is both accompanied and an electron. Similarly, something is a lonely positron iff it is both lonely and a positron.

and

Is either a lonely positron or an accompanied electron

$$= \{ \langle x, w \rangle \mid x \text{ is either a lonely positron or an accompanied electron at } w \}$$

$$= \{ \langle a, w_2 \rangle, \langle b, w_4 \rangle, \langle a, w_5 \rangle, \langle b, w_6 \rangle \}.$$

The instantiation of Is an electron and Is a proper part of through the space of worlds is described by Figure 1. The instantiation of Is either a lonely positron or an accompanied electron and Is a proper part of through the space of worlds is described by Figure 2. In these figures, an arrow going from one concrete thing to another indicates that the first thing is a proper part of the second thing at the relevant world, and a concrete thing appearing in a world indicates that it exists at that world. In Figure 1, a concrete thing is shaded in a world iff it is an electron at that world. In Figure 2, a concrete thing is shaded in a world iff it is either a lonely positron or an accompanied electron at that world. Note that Is an electron and Is either a lonely positron or an accompanied electron have the same pattern of instantiation through logical and mereological space in the sense that Figure 1 and Figure 2 can be transformed into each other by swapping around the worlds.

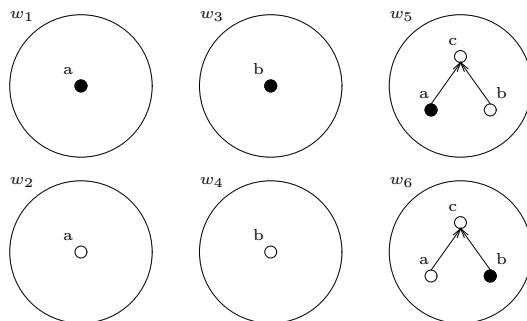


Figure 1: The instantiation of Is an electron

The properties Is an electron and Is either a lonely positron or an accompanied electron will be used to argue that ‘intrinsic’ cannot be defined using only broadly logical notions. To do this, I need to define the notion of a rigid formula. A one-place formula A with a free variable x is *rigid* iff $\lceil \forall x [\text{Necessarily}(A(x)) \vee \text{Necessarily}(\neg A(x))] \rceil$ is true. (In other words, a one-place formula is rigid iff it is satisfied by the same things at all worlds.) An example of a rigid formula is ‘ x is intrinsic’. This formula is rigid since, for each x , either it is necessary that x is intrinsic or it is necessary that x is not intrinsic. For example, Is an electron is necessarily intrinsic while Is either a lonely positron or an accompanied electron is necessarily not intrinsic. Another example of a rigid formula is the broadly logical formula ‘ x is instantiated by something at some world’. An example of a non-rigid

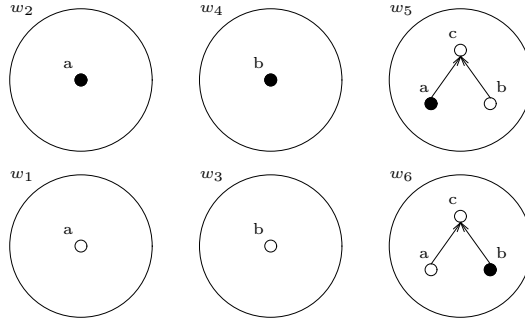


Figure 2: The instantiation of Is either a lonely positron or an accompanied electron

formula is the broadly logical formula ‘ x is instantiated by something’. This formula is non-rigid since there are properties like Is an electron that have instances but might have failed to do so.

Is an electron and Is either a lonely positron or an accompanied electron satisfy the same rigid broadly logical formulas. For example, they each satisfy the following formulas:

1. For some world w , at w , x is instantiated by something.
2. For some world w , at w , x is instantiated by something that is lonely.
3. For some world w , at w , x is instantiated by something that is accompanied.

Intuitively this is because whether or not a property satisfies a rigid broadly logical formula depends only on its pattern of instantiation through logical and mereological space. Since Is an electron and Is either a lonely positron or an accompanied electron have the same pattern of instantiation through logical and mereological space they therefore satisfy the same rigid broadly logical formulas.¹⁴ (A rigorous argument for this result is given in section 3.)

I can now show that ‘intrinsic’ cannot be defined using only broadly logical notions. Let us suppose that, on the contrary, ‘intrinsic’ can be defined using only broadly logical notions. Then ‘ x is intrinsic’ would be a broadly logical formula. Moreover, since ‘ x is intrinsic’ is a rigid formula, ‘ x is intrinsic’ would be a rigid broadly logical formula. Since Is an electron and Is either a lonely positron or an accompanied electron satisfy the same rigid broadly logical formulas, it would follow that either both of these properties satisfy ‘ x is intrinsic’ or both fail to satisfy ‘ x is intrinsic’. So Is an electron and Is either a

¹⁴Note that Is an electron and Is either a lonely positron or an accompanied electron do not satisfy all the same *non-rigid* broadly logical formulas. For example, let us suppose that w_1 is the actual world. Then Is an electron satisfies the non-rigid broadly logical formula ‘is instantiated by something’, whereas Is either a lonely positron or an accompanied electron does not.

lonely positron or an accompanied electron would either both be intrinsic or both fail to be intrinsic. However, Is an electron is intrinsic, whereas Is either a lonely positron or an accompanied electron is extrinsic. Hence, it is not possible to define ‘intrinsic’ using only broadly logical notions.

To simplify the argument, I have supposed that there are only three concrete things and six worlds. This simplification can be removed if Is an electron and Is a positron satisfy certain conditions, such as the condition that anything that can have one of these properties can have the other. (For a list of these conditions see (A1-9) in section 4.) Moreover, the argument can be made to work if we replace Is an electron and Is a positron with any pair of properties that satisfy these conditions. The argument that ‘intrinsic’ cannot be defined using only broadly logical notions, therefore, does not essentially depend on any assumptions about the particular properties Is an electron and Is a positron.

3 Isomorphisms

In this section I will define the notion of an isomorphism with respect to a set of relations. This notion will allow me to make precise the idea that Is an electron and Is either a lonely positron or an accompanied electron have the same “pattern of instantiation through logical and mereological space”. I will then state an important result involving these isomorphisms. This result will be used in section 4 to give a more rigorous and complete version of the argument that ‘intrinsic’ cannot be defined using only broadly logical notions.

A *permutation* on a set S is a one-to-one function from S onto itself.¹⁵ Let A be a set of relations. An A -isomorphism is a permutation on the set of worlds such that for any n -ary relation R in A , for any world w , and for any concrete things x_1, \dots, x_n , $\langle x_1, \dots, x_n \rangle$ instantiates R at w iff $\langle x_1, \dots, x_n \rangle$ instantiates R at $\sigma(w)$. Let \underline{E} be the property Is a concrete existent and \underline{P} be the 2-ary relation instantiated by $\langle x_1, x_2 \rangle$ at a world w iff, at w , x_1 exists, x_2 exists, and x_1 is a proper part of x_2 .¹⁶ A function σ is then a $\{\underline{E}, \underline{P}\}$ -isomorphism iff, for any world w , and for any concrete things x_1 and x_2 :

1. x_1 exists at w iff x_1 exists at $\sigma(w)$; and
2. (at w , x_1 exists, x_2 exists, and x_1 is a proper part of x_2) iff (at $\sigma(w)$, x_1 exists, x_2 exists, and x_1 is a proper part of x_2).

For an example of how this definition works, let us suppose that the set of concrete things

¹⁵A function f is a one-to-one function from a set D onto a set R iff i) no two distinct things are assigned by f to the same thing, ii) for each x in D , f assigns x to a member of R , iii) for each x not in D , f does not assign x to anything, and iv) for each y in R there is an x in D such that $f(x) = y$.

¹⁶Note that the predicate ‘exists’ does not express a property since, given the standard set-theoretical hierarchy of sets, there is no set containing all the existing things. In contrast, due to the assumption (3iv) in footnote 11, ‘is a proper part of ... at’ does express a relation.

and worlds is as it is described to be in section 2. Let p be the permutation on the set of worlds that assigns:

$$\begin{array}{ll} w_1 \Rightarrow w_2 & w_4 \Rightarrow w_3 \\ w_2 \Rightarrow w_1 & w_5 \Rightarrow w_5 \\ w_3 \Rightarrow w_4 & w_6 \Rightarrow w_6 \end{array}$$

It is easy to check that p is then a $\{\underline{E}, \underline{P}\}$ -isomorphism.

Define ‘ \cdot ’ to be an operator such that for any x , and for any permutation σ on the set of worlds:

1. if x is a world, then $(\sigma \cdot x) = \sigma(x)$;
2. if x is a concrete thing, then $(\sigma \cdot x) = x$;
3. if x is a set, then $(\sigma \cdot x) = \{(\sigma \cdot y) | y \in x\}$; and
4. if x is an ordered set $\langle x_1, \dots, x_n \rangle$, then $(\sigma \cdot x) = \langle (\sigma \cdot x_1), \dots, (\sigma \cdot x_n) \rangle$.¹⁷

The operator ‘ \cdot ’ can be used to make precise the notion of two properties “having the same pattern of instantiation through logical and mereological space” by defining F and G to have the same pattern of instantiation through logical and mereological space iff there is a $\{\underline{E}, \underline{P}\}$ -isomorphism σ such that $(\sigma \cdot F) = G$. It is easy to use this definition to show that, given that the set of concrete things and worlds is as it is described to be in section 2, Is an electron and Is either a lonely positron or an accompanied electron have the same pattern of instantiation through logical and mereological space. To do this, first suppose that the set of concrete things and worlds is as it is described to be in section 2. Then Is an electron is the set $\{\langle a, w_1 \rangle, \langle b, w_3 \rangle, \langle a, w_5 \rangle, \langle b, w_6 \rangle\}$, and Is either a lonely positron or an accompanied electron is the set $\{\langle a, w_2 \rangle, \langle b, w_4 \rangle, \langle a, w_5 \rangle, \langle b, w_6 \rangle\}$. It then follows that

$$\begin{aligned} (p \cdot \underline{\text{Is electron}}) &= \{(p \cdot \langle a, w_1 \rangle), (p \cdot \langle b, w_3 \rangle), (p \cdot \langle a, w_5 \rangle), (p \cdot \langle b, w_6 \rangle)\} \\ &= \{\langle (p \cdot a), (p \cdot w_1) \rangle, \langle (p \cdot b), (p \cdot w_3) \rangle, \langle (p \cdot a), (p \cdot w_5) \rangle, \langle (p \cdot b), (p \cdot w_6) \rangle\} \\ &= \{\langle a, p(w_1) \rangle, \langle b, p(w_3) \rangle, \langle a, p(w_5) \rangle, \langle b, p(w_6) \rangle\} \\ &= \{\langle a, w_2 \rangle, \langle b, w_4 \rangle, \langle a, w_5 \rangle, \langle b, w_6 \rangle\} \\ &= \underline{\text{Is either a lonely positron or an accompanied electron}}. \end{aligned}$$

Hence, Is an electron and Is either a lonely positron or an accompanied electron have the same pattern of instantiation through logical and mereological space, given the set of concrete things and worlds is as it is described to be in section 2.

¹⁷The third clause of this definition can be more formally written as: if x is a set then $(\sigma \cdot x) = \{y | \exists z(z \in x \text{ and } (\sigma \cdot z = y))\}$. Note that the third clause is redundant given the standard reduction of ordered sets to non-ordered sets, according to which any n -tuple $\langle x_1, \dots, x_n \rangle$ is identical to $\{\langle x_1, \dots, x_{n-1} \rangle, x_n\}$ and any 1-tuple $\langle x_1 \rangle$ is identical to $\{x_1\}$.

The following theorem describes an important feature of $\{\underline{E}, \underline{P}\}$ -isomorphisms. (A proof of this theorem is sketched in the Appendix.)

Theorem 1 *For any $\{\underline{E}, \underline{P}\}$ -isomorphism σ , and for any property F , F satisfies the same rigid broadly logical formulas as $(\sigma \cdot F)$.*

Theorem 1 will be used in the next section to argue that ‘intrinsic’ cannot be defined using only broadly logical notions. It can also be used to establish a claim made in section 2. Since $(p \cdot \text{Is electron})$ is identical to Is either a lonely positron or an accompanied electron if the set of concrete things and worlds is as it is described to be in section 2, it follows from Theorem 1 that Is an electron and Is either a lonely positron or an accompanied electron satisfy the same rigid broadly logical formulas if the set of concrete things and worlds is as it is described to be in section 2.

4 A Formal Argument

In arguing that ‘intrinsic’ cannot be defined using only broadly logical notions, I will make the following assumptions about the properties Is an electron and Is a positron:

- (A1) Is an electron and Is a positron are intrinsic properties.
- (A2) Necessarily, anything having either Is an electron or Is a positron is concrete.
- (A3) Is an electron and Is a positron are possibly instantiated: that is, it is possible for something to have Is an electron, and it is possible for something to have Is a positron.
- (A4) Is an electron and Is a positron are *incompatible* with each other: that is, it is impossible for something to have both these properties.
- (A5) Necessarily, anything having either Is an electron or Is a positron lacks proper parts.
- (A6) Necessarily, anything having either Is an electron or Is a positron exists: for any world w , if something is either an electron or a positron at w then it exists at w .
- (A7) Is an electron and Is a positron are *mutually accidental* properties: that is, anything that can have Is an electron can have Is a positron, and anything that can have Is a positron can have Is an electron.
- (A8) Is an electron and Is a positron are *independent of accompaniment*: that is, for any x , if there is a world at which x has Is an electron [Is a positron] then i) there is a world at which x is lonely and has Is an electron [Is a positron], and ii) there is a world at which x is accompanied and has Is an electron [Is a positron].¹⁸

¹⁸I am using the expression ‘independent of accompaniment’ differently to how Langton and Lewis use this expression in (Langton and Lewis 1998).

To state the final assumption, (A9), I need to define the notion of a qualitative property and the notion of an intrinsically qualitatively complete property. A *qualitative* property is intuitively a property that does not involve any particular individuals. For example, Is an electron and Is a positron that is one meter away from a positron are qualitative properties; whereas, Is an electron that is one meter away from Bill Clinton and Is either Bill Clinton or George Bush are non-qualitative properties. A property F is *intrinsically qualitatively complete* iff i) F is an intrinsic qualitative property, and ii) for any intrinsic qualitative property G , either F entails G , or F is incompatible with G . (A property F entails a property G iff, necessarily, anything having F has G .) Note that if something has an intrinsically qualitatively complete property, then what other intrinsic qualitative properties it has is determined by it having that property. So, if two things have the same intrinsically qualitatively complete property, then they share all the same intrinsic qualitative properties. In other words, things having the same intrinsically qualitatively complete property are exact duplicates of each other. The final assumption is:

(A9) Is an electron and Is a positron are intrinsically qualitatively complete properties.¹⁹

The truth of these assumptions is not uncontroversial. For example, while we have good grounds for believing that nothing is both an electron and a positron, it is doubtful whether we have good grounds for believing that, necessarily, nothing is both an electron and a positron. Hence, it is doubtful whether Is an electron is incompatible with Is a positron. The following argument, however, will work just as well if we replace Is an electron and Is a positron with any pair of properties that are intrinsic, possibly instantiated, incompatible with each other, mutually accidental, independent of accompaniment, intrinsically qualitatively complete, and are such that it is necessary that anything having them is concrete, exists and has no proper parts. The following argument therefore shows that if there are properties satisfying these conditions then ‘intrinsic’ cannot be defined using only broadly logical notions. Since there surely are such properties, the argument will show that ‘intrinsic’ cannot be defined using only these notions.²⁰

¹⁹Note that (A9) makes (A1) redundant.

²⁰(*) is still not wholly uncontroversial:

- (*) There are properties that are intrinsic, possibly instantiated, incompatible with each other, mutually accidental, independent of accompaniment, intrinsically qualitatively complete, and are such that it is necessary that anything having them is concrete, exists and has no proper parts.

Necessitarian molecularists, for example, reject (*) because they believe that (necessarily) everything has proper parts. These philosophers therefore deny that there are properties that are possibly instantiated and are such that it is necessary that anything having them has no proper parts. Radical essentialists also reject (*) because they believe that (necessarily) things have their properties essentially. Radical essentialists therefore deny that there are distinct properties that are mutually accidental. It therefore might be objected that the argument given here rests on controversial assumptions, and so is unconvincing. In response to this objection, first, the views of necessitarian molecularists and radical essentialists are *prima facie* implausible, and few philosophers endorse them. (*) is therefore plausibly true and so—provided the argument given here is correct—‘intrinsic’ plausibly cannot be defined using only broadly

The argument to be given is summarised as follows:

- (P1) $S1$ is true
(P2) If $S1$ is true then $S2$ is true
(P3) If $S2$ is true then $S3$ is true

(C) $S3$ is true

where $S1$, $S2$ and $S3$ are the sentences:

- $S1$ For any x , either there are no worlds at which x is a lonely electron and no worlds at which x is a lonely positron, or there is a unique world at which x is a lonely electron and a unique world at which x is a lonely positron.
- $S2$ Is an electron is intrinsic, Is either a lonely positron or an accompanied electron is extrinsic, and there is a $\{\underline{E}, \underline{P}\}$ -isomorphism σ such that $(\sigma \cdot \underline{\text{Is an electron}})$ is identical to Is either a lonely positron or an accompanied electron.
- $S3$ ‘Intrinsic’ cannot be defined using only broadly logical notions.

The conclusion of this argument is that ‘intrinsic’ cannot be defined using only broadly logical notions. The argument is clearly valid. Hence, to show that ‘intrinsic’ cannot be defined using only broadly logical notions, I need to show that each of the argument’s premises are true. I will consider each premise in turn.

4.1 The First Premise

(P1) asserts that $S1$ is true, where $S1$ is the sentence:

- $S1$ For any x , either there are no worlds at which x is a lonely electron and no worlds at which x is a lonely positron, or there is a unique world at which x is a lonely electron and a unique world at which x is a lonely positron.

Since Is an electron and Is a positron are mutually accidental, for any x , there is a world at which x is an electron iff there is a world at which x is a positron. Since Is an electron and Is a positron are independent of accompaniment, it follows that:

logical notions without relying on special assumptions about properties or worlds. Second, modified versions of the argument presented here can be given that are acceptable to necessitarian molecularists and radical essentialists. (For further discussion see (Marshall ms).) Third, it would clearly be desirable to give a definition of ‘intrinsic’ that is compatible with standard views of mereology and modality according to which (*) is true (and is compatible with standard views about the nature of properties and worlds). The argument given here shows that this is not possible.

- (I) For any x , there is a world at which x is a lonely electron iff there is a world at which x is a lonely positron.

I will now argue that (I) entails $S1$ by using the assumption that Is an electron and Is a positron are intrinsically qualitatively complete properties. I will first argue that, for any x , if x is a lonely electron at worlds w and w' , then x has the same properties at w and w' . First, since Is an electron is intrinsically qualitatively complete, x being an electron necessitates which intrinsic qualitative properties x has. So, x being an electron necessitates that x has a certain size, mass, charge and so on. Second, since x being an electron necessitates which intrinsic qualitative properties x has, x being a lonely electron necessitates both which intrinsic qualitative properties x has and which extrinsic qualitative properties x has. So, x being a lonely electron not only entails that x has a certain size, mass and charge, it also entails that x is not one meter away from any cube, is the only electron, is the most charged particle, and so on. Third, since x being a lonely electron necessitates which qualitative properties x has, and x being a lonely electron entails that x is the only existing concrete thing, x being a lonely electron necessitates all of the properties x has.²¹ For example, x being a lonely electron necessitates that x is identical to x , that x does not have George Bush as a proper part, and that x is not one meter away from George Bush.²² Hence, for any x , if x is a lonely electron at worlds w and w' , then x has the same properties at w that it has at w' .

Now suppose that, for some x , x is a lonely electron at worlds w and w' . If $w \neq w'$ then it would follow from the theory of properties assumed in section 1 that x has the property $\{< x, w >\}$ at w , but does not have the property $\{< x, w >\}$ at w' . Since x has the same properties at w and w' , it follows that $w = w'$. Hence:

- (II) For any x , if there is a world at which x is a lonely electron then it is the unique world at which x is a lonely electron.

²¹The following is a proof that if something is either a lonely electron or a lonely positron at a world then it is the only existing concrete thing at that world: By the definition of 'is lonely', an x is lonely at a world w iff x does not coexist with any contingent thing wholly distinct from itself at w . It follows from the assumptions made in footnote 11 that x is contingent at w iff x is a concrete existent at w . Hence, if x is lonely at w then x does not coexist with any concrete thing wholly distinct from itself at w . It follows from (A2, A5-6) that if x is either an electron or a positron at w then x exists, is concrete, and has no proper parts at w . Hence, if an x is either a lonely electron or a lonely positron at w , then x is the only existing concrete thing at w .

²²The general principle underlying this argument is that how things are is necessitated by what concrete things exist, what intrinsic qualitative properties these concrete things have, and what intrinsic qualitative relations these concrete things stand in to each other. (Intuitively, an n -ary relation R is intrinsic iff, necessarily, for any x_1, \dots, x_n , if $\langle x_1, \dots, x_n \rangle$ instantiates R then $\langle x_1, \dots, x_n \rangle$ instantiates R in virtue of how x_1, \dots, x_n are and how they are related to each other.) Note that it follows from this principle that how an abstract thing is (both intrinsically and extrinsically) at a world w cannot be different from how it is (both intrinsically and extrinsically) at a world w' without there being a difference in what concrete things exist, what intrinsic qualitative properties these concrete things have, or what intrinsic qualitative relations these concrete things stand in to each other at w and w' .

A similar argument shows that

- (III) For any x , if there is a world at which x is a lonely positron then it is the unique world at which x is a lonely positron.

It follows from (I-III) that for any x , either there are no worlds at which x is a lonely electron and no worlds at which x is a lonely positron, or there is a unique world at which x is a lonely electron and a unique world at which x is a lonely positron. This establishes $S1$ and (P1).

4.2 The Second Premise

(P2) asserts that if $S1$ is true then $S2$ is true, where $S1$ and $S2$ are the sentences:

$S1$ For any x , either there are no worlds at which x is a lonely electron and no worlds at which x is a lonely positron, or there is a unique world at which x is a lonely electron and a unique world at which x is a lonely positron.

$S2$ Is an electron is intrinsic, Is either a lonely positron or an accompanied electron is extrinsic, and there is a $\{\underline{E}, \underline{P}\}$ -isomorphism σ such that $(\sigma \cdot \underline{\text{Is an electron}})$ is identical to Is either a lonely positron or an accompanied electron.

Let us suppose that $S1$ is true. Define σ to be the function on the set of worlds such that:

1. if w is a world at which there is a lonely electron x , then $\sigma(w)$ is the unique world at which x is a lonely positron;
2. if w is a world at which there is a lonely positron x , then $\sigma(w)$ is the unique world at which x is a lonely electron; and
3. if w is a world at which there are no lonely electrons or lonely positrons, then $\sigma(w) = w$.

The function σ is well-defined since there is at most one lonely electron or lonely positron at each world. (This is because if something is either a lonely electron or a lonely positron at a world then it is the only existing concrete thing at that world.²³) It is straightforward to verify that σ is a $\{\underline{E}, \underline{P}\}$ -isomorphism.²⁴

²³See footnote 21 for a proof of this claim.

²⁴The proof that σ is a $\{\underline{E}, \underline{P}\}$ -isomorphism is straightforward but somewhat lengthy, and requires the background assumptions made in section 1 and footnote 11.

For an illustration of this definition, let us suppose that the set of concrete things and worlds is as it is described to be in section 2. Then i) since w_1 is the unique world at which a is a lonely electron, and w_2 is the unique world at which a is a lonely positron, σ maps w_1 to w_2 and w_2 to w_1 ; ii) since w_3 is the unique world at which b is a lonely electron, and w_4 is the unique world at which b is a lonely positron, σ maps w_3 to w_4 and w_4 to w_3 ; and iii) since there are no lonely electrons or positrons at w_5 or w_6 , σ maps these worlds to themselves. It follows that, if the set of worlds and concrete things is as it is described to be in section 2, σ is the function p defined in section 3.

I will now show that $(\sigma \cdot \text{Is an electron})$ is identical to Is either a lonely positron or an accompanied electron. Let $K = (\sigma \cdot \text{Is an electron})$. Then,

$$\begin{aligned} K &= (\sigma \cdot \{ \langle x, w \rangle \mid x \text{ is an electron at } w \}) \\ &= \{ \langle (\sigma \cdot x), (\sigma \cdot w) \rangle \mid x \text{ is an electron at } w \}. \end{aligned}$$

For any x and w , if x is an electron at w , then x is concrete and w is a world.²⁵ Since $(\sigma \cdot x) = x$ if x is concrete, and $(\sigma \cdot w) = \sigma(w)$ if w is a world, it follows that

$$K = \{ \langle x, \sigma(w) \rangle \mid x \text{ is an electron at } w \}.$$

Let $K_1 = \{ \langle x, \sigma(w) \rangle \mid x \text{ is a lonely electron at } w \}$. Since σ maps the set of worlds at which there is a lonely electron onto the set of worlds at which there is a lonely positron,

$$K_1 = \{ \langle x, w \rangle \mid x \text{ is a lonely positron at } w \}.$$

Let $K_2 = \{ \langle x, \sigma(w) \rangle \mid x \text{ is an accompanied electron at } w \}$. If something is an accompanied electron at a world then there are no lonely electrons or lonely positrons at that world. Since $\sigma(w) = w$ if w is a world at which there are no lonely electrons or lonely positrons,

$$K_2 = \{ \langle x, w \rangle \mid x \text{ is an accompanied electron at } w \}.$$

Since $K = K_1 \cup K_2$, it follows that

$$\begin{aligned} K &= \{ \langle x, w \rangle \mid x \text{ is a lonely positron at } w \} \cup \{ \langle x, w \rangle \mid x \text{ is an accompanied electron at } w \} \\ &= \{ \langle x, w \rangle \mid \text{at } w, x \text{ is either a lonely positron or an accompanied electron} \} \\ &= \underline{\text{Is either a lonely positron or an accompanied electron}}. \end{aligned}$$

Hence $(\sigma \cdot \text{Is an electron})$ is identical to Is either a lonely positron or an accompanied electron. Since Is either a lonely positron or an accompanied electron is extrinsic, this establishes (P2).

(The fact that Is either a lonely positron or an accompanied electron is extrinsic can be derived from the assumptions (A3-4, A8-9) as follows: Since Is a positron is a possibly

²⁵See footnote 10.

instantiated property that is independent of accompaniment, for some x , and for some worlds w_1 and w_2 i) at w_1 , x is lonely and has Is a positron; and ii) at w_2 , x is accompanied and has Is a positron. Since Is a positron is intrinsically qualitatively complete, x has the same intrinsic qualitative properties at w_1 and w_2 . Since Is an electron and Is a positron are incompatible with each other, x has the qualitative property Is either a lonely positron or an accompanied electron at w_1 but does not have it at w_2 . (Is either a lonely positron or an accompanied electron is a qualitative property since Is an electron, Is a positron, Is lonely and Is accompanied are qualitative properties, and conjunctions and disjunctions of qualitative properties are themselves qualitative. Is an electron and Is a positron are qualitative since they are qualitatively intrinsically complete.) Since x has the same intrinsic qualitative properties at w_1 and w_2 , and x has the qualitative property Is either a lonely positron or an accompanied electron at w_1 , but not at w_2 , Is either a lonely positron or an accompanied electron is not intrinsic. Hence Is either a lonely positron or an accompanied electron is extrinsic.)

4.3 The Third Premise

(P3) asserts that if $S2$ is true then $S3$ is true, where $S2$ and $S3$ are the sentences:

$S2$ Is an electron is intrinsic, Is either a lonely positron or an accompanied electron is extrinsic, and there is a $\{\underline{E}, \underline{P}\}$ -isomorphism σ such that $(\sigma \cdot \underline{\text{Is an electron}})$ is identical to Is either a lonely positron or an accompanied electron.

$S3$ ‘Intrinsic’ cannot be defined using only broadly logical notions.

Let us suppose that $S2$ is true. As noted in section 2, ‘ x is intrinsic’ is a rigid formula. Hence, if ‘intrinsic’ could be defined using only broadly logical notions, ‘ x is intrinsic’ would abbreviate a rigid broadly logical formula. It follows from Theorem 1 that Is an electron and Is either a lonely positron or an accompanied electron would then either both satisfy ‘ x is intrinsic’ or both fail to satisfy ‘ x is intrinsic’. So, Is an electron and Is either a lonely positron or an accompanied electron would either both be intrinsic or both fail to be intrinsic. However, Is an electron is intrinsic while Is either a lonely positron or an accompanied electron is extrinsic. Hence, ‘intrinsic’ cannot be defined using only broadly logical notions. This establishes (P3).

5 Extensions to the Argument

The argument given in section 4 can be extended to show that it is not possible to define ‘intrinsic’ in terms of certain larger sets of notions without relying on special assumptions about properties or worlds. For example, the argument can be extended to show that it is

not possible to define ‘intrinsic’ using only broadly logical notions and basic spatiotemporal notions without relying on such assumptions. The argument can also be extended to show that it is not possible to define ‘intrinsic’ using only broadly logical notions, basic spatiotemporal notions, and the notion of a qualitative property without relying on these assumptions. Further extensions of the argument can also be made.²⁶

I will describe one of these ways of extending the argument by sketching the argument that ‘intrinsic’ cannot be defined using only broadly logical and basic spatiotemporal notions, where a basic spatiotemporal notion is one that expresses a fundamental spatiotemporal property or relation. To give this argument, I will need to assume some specific theory of space and time. For this purpose, I will assume the following theses about space and time:

- (ST1) The fundamental spatiotemporal properties and relations are 2-ary relations like Is 1 meter from and Is 10 seconds after.²⁷
- (ST2) For any fundamental 2-ary spatiotemporal relation R , for any x and y , and for any world w , $\langle x, y \rangle$ instantiates R at w iff x and y are concrete existents at w .
- (ST3) Necessarily, for any x , x does not stand in any fundamental spatiotemporal relation to itself.

The following argument relies on the assumptions (ST1-3) and (A1-9). The argument can be modified to work with other theories of space and time.

Let S be the set of 2-ary fundamental spatiotemporal relations, and let $S^* = \{\underline{E}, \underline{P}\} \cup S$. A function σ is then a S^* -isomorphism iff σ is a permutation on the set of worlds such that, for each concrete thing x_1 and x_2 , and for each world w :

1. x_1 exists at w iff x_1 exists at $\sigma(w)$;
2. (at w , x_1 exists, x_2 exists, and x_1 is a proper part of x_2) iff (at $\sigma(w)$, x_1 exists, x_2 exists, and x_1 is a proper part of x_2); and
3. for each fundamental 2-ary spatiotemporal relation R , $\langle x_1, x_2 \rangle$ instantiates R at w iff $\langle x_1, x_2 \rangle$ instantiates R at $\sigma(w)$.

The argument that ‘intrinsic’ cannot be defined using only broadly logical and basic spatiotemporal notions is summarised as follows:

²⁶For further discussion of these extensions see (Marshall ms).

²⁷I am ignoring complications arising from the special and general theories of relativity.

(P1')	$S1'$ is true
(P2')	If $S1'$ is true then $S2'$ is true
(P3')	If $S2'$ is true then $S3'$ is true
<hr/>	
(C')	$S3'$ is true

where $S1'$, $S2'$ and $S3'$ are the sentences:

$S1'$ For any x , either there are no worlds at which x is a lonely electron and no worlds at which x is a lonely positron, or there is a unique world at which x is a lonely electron and a unique world at which x is a lonely positron.

$S2'$ Is an electron is intrinsic, Is either a lonely positron or an accompanied electron is extrinsic, and there is a S^* -isomorphism σ such that $(\sigma \cdot \text{Is an electron})$ is identical to Is either a lonely positron or an accompanied electron.

$S3'$ 'Intrinsic' cannot be defined using only broadly logical and basic spatiotemporal notions.

Since $S1'$ is identical to $S1$, I only need to discuss arguments for the premises (P2') and (P3'). The required argument for (P2') is effectively the same as the argument given for (P2) in section 4, except that it needs to be shown that the permutation σ defined in section 4 is a S^* -isomorphism. The required argument for (P3') is effectively the same as the argument given for (P3) in section 4 except that Theorem 1' must be used in the place of Theorem 1. (The proof of Theorem 1' is similar to the proof of Theorem 1.)

Theorem 1' *For any S^* -isomorphism σ , and for any property F , F satisfies the same rigid formulas containing only broadly logical vocabulary and basic spatiotemporal predicates as $(\sigma \cdot F)$.*

6 Further Applications

The type of argument given in section 4 can be applied to notions other than that of an intrinsic property. For example, it can be used to show that it is not possible to give a broadly logical definition of any of the following notions without relying on special assumptions about properties or worlds: the notion of a qualitative property, the notion of a perfectly natural property, the notion of a disjunctive property, and the notion of a positive property.

Intuitively, a perfectly natural property is a fundamental property; a disjunctive property is a property that does not mark out a respect in which its instances are similar to

each other; and a positive property is a property things have in virtue of how they are, as opposed to how they fail to be. Examples of perfectly natural properties possibly include Has unit positive charge and Has unit mass. Examples of properties that are not perfectly natural plausibly include Is an electron and Is either a lonely positron or an accompanied electron. Has unit positive charge and Is an electron are plausibly non-disjunctive properties, whereas Is either a lonely positron or an accompanied electron is disjunctive. Has unit positive charge and Is an electron are plausibly positive properties, whereas Is not one meter away from something and Is either a lonely positron or an accompanied electron are plausibly not positive properties. (Is either a lonely positron or an accompanied electron is non-positive because Is lonely is a negative property: that is, it is a property something has in virtue of how it fails to be, rather than how it is.)²⁸

The following is a quick and rough argument that none of these notions can be defined using only broadly logical notions. Roughly, ‘intrinsic’ cannot be defined using only broadly logical notions because Is an electron and Is either a lonely positron or an accompanied electron satisfy the same rigid broadly logical formulas, but differ with respect to their intrinsicness. Now, Is an electron is both non-disjunctive and positive, whereas Is either a lonely positron or an accompanied electron is both disjunctive and non-positive. Since Is an electron and Is either a lonely positron or an accompanied electron satisfy the same rigid broadly logical formulas, but differ with respect to their disjunctiveness and their positivity, neither ‘is disjunctive’ or ‘is positive’ can be defined using only broadly logical notions. Neither Is an electron or Is either a lonely positron or an accompanied electron are perfectly natural. So, to show that ‘is perfectly natural’ cannot be defined using only broadly logical notions, we need to replace this pair of properties with another pair of properties, one of which is perfectly natural and the other of which is not perfectly natural. Let N and N' be perfectly natural properties that are possibly instantiated, incompatible with each other, mutually accidental, independent of accompaniment, intrinsically qualitatively complete, and such that it is necessary that anything having them is concrete, exists and has no proper parts. Then, by the argument given in section 4, N will satisfy the same rigid broadly logical formulas as Is either lonely and has N' , or accompanied and has N . Since N is perfectly natural and Is either lonely and has N' , or accompanied and has N is not perfectly natural, ‘is perfectly natural’ cannot be defined using only broadly logical notions. For the case of qualitative properties, we replace the property Is either a lonely positron or an accompanied electron with the property Is either a lonely positron that is identical to a , or a lonely electron that is not identical to a , or an accompanied electron, where ‘ a ’ is the name of something that is an electron at some world. By using an argument similar to the one given in section 4, we can show that Is an electron and Is either a lonely positron that is identical to a , or a lonely electron that is not identical to a , or an accompanied electron satisfy the same rigid broadly logical formulas. Since Is an electron is qualitative, whereas Is either a lonely positron that is identical to a , or a lonely electron that is not identical to a , or an accompanied electron is non-qualitative, it follows that ‘is qualitative’ cannot

²⁸For further discussion of these notions see (Lewis 1983b), (Langton and Lewis 1998) and (Marshall ms).

be defined using only broadly logical notions.

As in the case of intrinsic properties, these arguments can be extended to show that these notions cannot be defined in terms of certain larger sets of notions. For example, it can be shown that none of these notions can be defined using only broadly logical and basic spatiotemporal notions.

7 Conclusion

I have argued that it is not possible to define ‘intrinsic’ using only broadly logical notions without relying on special assumptions about properties or worlds. I have also indicated how this argument can be extended to show that ‘intrinsic’ cannot be defined using only broadly logical and basic spatiotemporal notions without relying on special assumptions about these entities, and I have claimed that the argument can be further extended to show that ‘intrinsic’ cannot be defined using only broadly logical notions, basic spatiotemporal notions and the notion of a qualitative property without relying on these assumptions. Finally, I have indicated how this type of argument can also be used to show that it is not possible to define either the notion of a qualitative property, the notion of a perfectly natural property, the notion of a disjunctive property, or the notion of a positive property using only broadly logical notions without relying on these assumptions.

If we cannot define ‘intrinsic’ using only broadly logical notions (or using only broadly logical notions, basic spatiotemporal notions and the notion of a qualitative property) we might try to define ‘intrinsic’ using other notions (perhaps in conjunction with broadly logical notions, basic spatiotemporal notions and the notion of a qualitative property). In particular, we might try to define ‘intrinsic’ using either the notion of a perfectly natural property, the notion of a disjunctive property, or the notion of a positive property. The argument in this paper cannot be applied to rule out definitions of ‘intrinsic’ involving these notions. This is roughly because, while Is an electron and Is either a lonely positron or an accompanied electron satisfy the same rigid formulas containing broadly logical vocabulary, they do not satisfy the same rigid formulas involving the predicates ‘is a perfectly natural property’, ‘is a disjunctive property’ and ‘is a positive property’. For example, Is either a lonely positron or an accompanied electron plausibly satisfies the rigid formula ‘ x is a disjunctive property’, whereas Is an electron plausibly does not.

Arguably, neither the notion of a perfectly natural property, nor the notion of a disjunctive property, nor the notion of a positive property are as well understood as any of the broadly logical notions. Indeed, it is plausible that they are no better understood than the notion of an intrinsic property which we are trying to define, or the notion of a thing having a property wholly in virtue of how it is which was used to define ‘intrinsic’ at the beginning of this paper. These notions, however, are not closely related to the notion of an intrinsic property; they are further removed from the notion of an intrinsic property than

is, for example, the notion of a thing having a property wholly in virtue of how it is. While a definition of ‘intrinsic’ involving these notions would perhaps not be as illuminating as one involving only broadly logical notions, a successful definition involving these notions would still be welcome.²⁹

Appendix

Theorem 1 can be proven by first proving Theorems A-D. The proofs of these theorems rely on the assumptions made in section 1 and footnote 11, and on the mereological principle (%).

(%) For any x and y , and for any worlds w and w' :

1. If x exists at w and w' , and y is non-existent at w and w' , then x is a proper part of y at w iff x is a proper part of y at w' .
2. If x and y are non-existent at w and w' , then x is a proper part of y at w iff x is a proper part of y at w' .
3. If x is non-existent at w , and y exists at w , then x is not a proper part of y at w .

It follows from (%) that a function σ is a $\{\underline{E}, \underline{P}\}$ -isomorphism iff σ is a permutation on the set of worlds such that for each concrete thing x_1 and x_2 , and for each world w :

1. x_1 exists at w iff x_1 exists at $\sigma(w)$; and
2. x_1 is a proper part of x_2 at w iff x_1 is a proper part of x_2 at $\sigma(w)$.

Theorem A *Suppose σ is a $\{\underline{E}, \underline{P}\}$ -isomorphism. Then, for any x_1, x_2 and x_3 : i) x_1 is a world iff $(\sigma \cdot x_1)$ is a world; ii) x_1 exists at x_3 iff $(\sigma \cdot x_1)$ exists at $(\sigma \cdot x_3)$; iii) x_1 is a proper part of x_2 at x_3 iff $(\sigma \cdot x_1)$ is a proper part of $(\sigma \cdot x_2)$ at $(\sigma \cdot x_3)$; iv) x_1 is a set iff $(\sigma \cdot x_1)$ is a set; and v) $x_1 \in x_2$ iff $(\sigma \cdot x_1) \in (\sigma \cdot x_2)$.*

Proof. The proof is straightforward, although relatively tedious.

Let ‘ \circ ’ be an operator such that, for any permutation σ on the set of worlds, and for any function f , $(\sigma \circ f)$ is the function that assigns each x in the domain of f to $(\sigma \cdot f(x))$.

Theorem B *Suppose σ is a $\{\underline{E}, \underline{P}\}$ -isomorphism, and A is a formula in L . Then, for any assignment a to the variables in L , and for any world w , A is true _{a,w} iff A is true _{$(\sigma \circ a), (\sigma \cdot w)$} .*

²⁹For proposed definitions of ‘intrinsic’ that involve these notions see (Lewis 1983b), (Langton and Lewis 1998), (Marshall and Parsons 2001) and (Marshall ms).

Proof. The proof is by induction on the number of operators in A .

Theorem C *Suppose σ is a $\{\underline{E}, \underline{P}\}$ -isomorphism, and A is a rigid formula in L . Then, for any assignment a to the variables of L , A is true_a iff A is $\text{true}_{(\sigma \circ a)}$.*

Proof. It follows from the account of the truth conditions of L assumed in footnote 11 that (1) A is true_a iff A is $\text{true}_{a, \text{the actual world}}$, and (2) A is $\text{true}_{(\sigma \circ a)}$ iff A is $\text{true}_{(\sigma \circ a), \text{the actual world}}$. Since A is a rigid formula, (3) A is $\text{true}_{(\sigma \circ a), \text{the actual world}}$ iff A is $\text{true}_{(\sigma \circ a), (\sigma \cdot (\text{the actual world}))}$. From Theorem B, (4) A is $\text{true}_{(\sigma \circ a), (\sigma \cdot (\text{the actual world}))}$ iff A is $\text{true}_{a, \text{the actual world}}$. It follows from (1 – 4) that A is true_a iff A is $\text{true}_{(\sigma \circ a)}$. This establishes Theorem C.

Theorem D *For any $\{\underline{E}, \underline{P}\}$ -isomorphism σ , and for any x , x satisfies the same rigid broadly logical formulas as $(\sigma \cdot x)$.*

Proof. Let x be anything at all. Let σ be a $\{\underline{E}, \underline{P}\}$ -isomorphism, A be a rigid formula in L containing one free variable X , and a be an assignment to the variables in L that assigns X to x . Then (1) x satisfies A iff A is true_a , and (2) $(\sigma \cdot x)$ satisfies A iff A is $\text{true}_{(\sigma \circ a)}$. It follows from (1), (2) and Theorem C, that x satisfies A iff $(\sigma \cdot x)$ satisfies A . This establishes Theorem D.

Theorem 1 is an obvious corollary of Theorem D.

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