

Chance

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Much is asked of the concept of *chance*. It has been thought to play various roles, some in tension with or even incompatible with others. Chance has been characterized negatively, as the absence of causation; yet also positively—the ancient Greek *τυχη* reifies it—as a cause of events that are not governed by laws of nature, or as a feature of the laws themselves. Chance events have been understood epistemically as those whose causes are unknown; yet also objectively as a distinct ontological kind, sometimes called ‘pure’ chance events. Chance gives rise to individual unpredictability and disorder; yet it yields collective predictability and order—stable long-run statistics, and in the limit, aggregate behavior susceptible to precise mathematical theorems. Some authors believe that to posit chances is to abjure explanation; yet others think that chances are themselves explanatory. During the Enlightenment, talk of ‘chance’ was regarded as unscientific, unphilosophical, the stuff of superstition or ignorance; yet today it is often taken to be a fundamental notion of our most successful scientific theory, quantum mechanics, and a central concept of contemporary metaphysics.

Chance has both negative and positive associations in daily life. The old word in English for it, *hazard*, which derives from French and originally from Arabic, still has unwelcome connotations of risk; ‘chance’ evokes uncertainty, uncontrollability, and chaos. Yet chance is also allied with luck, fortune, freedom from constraint, and diversity. And it apparently has various practical uses and benefits. It forms the basis of randomized trials in statistics, and of mixed strategies in decision theory and game theory; it is appealed to in order to resolve problems of fair division and other ethical

stalemates; and it is even thought to underpin biological and cultural adaptation. Throughout history, ‘chance’ devices have been a source of entertainment, as well as of scorn.

A brief history of ‘chance’

The study of gambling games motivated the first serious mathematical study of chance by Pascal and Fermat in the mid-17th century, culminating in the *Port-Royal Logic*. But inchoate ideas about chance date back to antiquity. Epicurus, and later Lucretius, believed that atoms occasionally underwent uncaused, indeterministic swerves—an early doctrine of pure chances. Aristotle, by contrast, believed in the necessity of all events, regarding what we call coincidences—as in ‘we met at the market place by chance’—as intersections of independent deterministic causal chains, a view later shared by Aquinas, Cournot, and Mill. Augustine believed that God’s will controls everything, and thus that nothing happens by chance. In the middle ages, Averroes had a notion of ‘equipotency’ that arguably resonated with Leibniz’s and later Laplace’s ideas about ‘equipossibility’, which undergirded their ‘classical’ interpretation of probability: the probability of an event is the ratio of the number of ‘equipossible’ cases in which it occurs to the total number of such cases. Cardano, Galileo, Fermat and Pascal also anticipated this interpretation.

Throughout the development of probability theory during the 17th-19th centuries by authors such as Huygens, Bernoulli, Bayes, Laplace, Condorcet, de Moivre, and Venn, the fortunes of ‘chance’ were at best mixed. De Moivre called chance “a mere word”. Hume captured the attitude of his time: “’tis commonly allowed by philosophers that what the vulgar call chance is nothing but a secret and conceal’d cause”. The triumphs of Newtonian mechanics engendered a supreme confidence in determinism, personified by

Laplace's image of an intelligent being (the so-called 'Laplacean demon') for whom "nothing would be uncertain and the future, as the past, would be present to its eyes." Eliminativism about chance in nature had, moreover, good theological credentials: God's omniscience apparently made the world safe for determinism. But even the atheistic Russell insisted that a chance event is merely one whose cause is unknown. Bradley found the very notion of chance unintelligible.

Nonetheless, other intellectual developments set the stage for a revival of chance. With the burgeoning of social statistics in the 19th century came a realization that various social phenomena—births, deaths, crime rates, etc.—while unpredictable on an individual basis, conformed to large-scale statistical regularities. A somewhat analogous pattern of collective-order-from-individual-chaos appeared in statistical mechanics. The social sciences and then the physical sciences thus admitted statistical laws into their conceptual repertoire. This culminated in the early 20th century with the advent of quantum mechanics (QM), which appeared to show that chance was irreducible and ineradicable. Kolmogorov's axiomatization of probability arrived on the heels of Heisenberg and Schrödinger bringing QM to its apogee.

Meanwhile, chance was also making a comeback in philosophy. Peirce defended pure chances on the basis of empirical evidence. James saw the postulation of chances as a way to resolve the apparent conflict between determinism and free will. To be sure, philosophers such as Mill, Schlick and Broad thought that the capriciousness of chance could provide no ground for genuine freedom. Nevertheless, chance had regained its respectability. By the 1950's, Reichenbach's work on probabilistic causation placed chance in the limelight in the philosophy of science.

The mathematics of chance

The mathematics of chance, unlike its philosophy, is relatively uncontroversial. That mathematics is widely taken to be *probability theory*. In Kolmogorov's theory, 'events' are assigned numerical values that are non-negative; their maximum value is 1; and the probability of at least one of two mutually exclusive events occurring is the sum of their probabilities. Axiomatically:

$$P(X) \geq 0;$$

$$P(\Omega) = 1, \text{ where } \Omega \text{ is the 'universal set' of all possible outcomes;}$$

$$P(X \cup Y) = P(X) + P(Y) \text{ if } X \cap Y = \emptyset. \text{ (This has an infinite generalization.)}$$

The *conditional probability* of A given B is:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad (P(B) > 0).$$

While Kolmogorov's theory remains the orthodoxy, some philosophers (e.g. Fetzer, Humphreys, Popper) question its appropriateness for chance.

Chance in science

Probability was introduced into physics in the late 19th century, when Maxwell and Boltzmann grounded thermodynamics in statistical mechanics. The status of this probability was an important interpretive issue, but it was not universally regarded as objective chance. Statistical mechanics was based on Newtonian particle mechanics, which was apparently deterministic. There are profound and ongoing controversies over the existence and nature of chances in both statistical mechanics and quantum mechanics.

In non-relativistic QM, according to the canonical "Copenhagen" interpretation, there are two rules for the evolution of a physical system:

1. *Schrödinger's equation* prescribes a deterministic evolution for the system's state.

Typically the state is a combination—"superposition"—of the various definite property

states that the system might possess—e.g., definite position, definite momentum, etc. While the system is in a superposition, it has no single value for such quantities.

2. The *collapse postulate* is where chance enters QM. Upon measurement of such a quantity, the state instantaneously collapses to one of the quantity's "eigenstates"; which one is a matter of chance, the probability for each being derivable by Born's rule.

Einstein considered this intrusion of chance into micro-physics an unacceptable violation of causality, and hoped for an underlying deterministic theory with "hidden" variables that explains the apparently-chancy behavior of quantum systems. In 1935, Einstein, Podolsky and Rosen (EPR) insisted that there must be such an underlying theory, arguing that the QM description of a certain two-particle system is incomplete. Bohr and Heisenberg criticized the EPR argument effectively, and since an experimental test of an EPR-pair of particles appeared to be physically unrealizable, most physicists quickly forgot the debate.

In 1952, Bohm proposed a variant of the EPR setup using two "entangled" particles with correlated spins. Bohm's variant was both immune to the Bohr/Heisenberg criticisms and physically realizable. In 1965, Bell proved a now-legendary theorem stating that no local hidden-variable theory, of the type desired by Einstein, could replicate the statistical predictions of QM for the correlated spins. Contrary to what EPR had assumed, an underlying hidden-variable theory that assigned definite local values of spin to individual particles was incompatible with QM's predictions. Physicists realized then that a decisive experimental test was possible, and numerous experiments were performed in the 1970s, culminating in Aspect's 1982 experiments, widely regarded as decisive. Nature sided with Bohr and Heisenberg, not Einstein.

Ironically, however, this confirmation of QM's predictions did not definitively show that God does play dice. In 1952, Bohm had also formulated a hidden-variable variant of

QM that ascribes definite positions to all particles at all times, reproduces all the experimental predictions of standard QM, and is perfectly deterministic! This is consistent with Bell's theorem. No *local* hidden-variable theory could match QM's predictions for entangled particles, but Bohm's version of QM is *non-local*: a particle in one place may be affected, instantaneously, by distant events. Einstein would have approved of Bohm's theory for its deterministic microphysics and disapproved of it for violating the even more cherished precept of locality.

There are other versions of QM besides Bohm's that reject chancy collapses. It is thus unclear whether the success of quantum mechanical theories implies a fundamental indeterminism in nature, and whether future experiments can resolve the issue.

Evolutionary biology is another area of science in which the existence and role of chance has been sharply debated. Evolutionary fitness is held by some philosophers and biologists to be fundamentally chancy, while others disagree.

Philosophical accounts of chance

Nowadays, 'chance' is typically taken to be synonymous with 'objective probability', distinguished from epistemic or subjective probability. *Frequentists*, originating with Venn, identify chance with relative frequency—e.g., the chance that a particular coin lands heads is the frequency of tosses on which it so lands, divided by the total number of tosses. If we restrict ourselves to actual outcomes, then such frequencies will presumably be finite. A concern is that the outcomes may ill-reflect the true chances; a fair coin may land heads nine times out of ten. At an extreme, the *problem of the single case*, various events are unrepeatable, yet arguably have non-trivial chances—e.g., the next presidential election's outcome; then such mismatch between chance and relative frequency is guaranteed. Sometimes we might include in the reference class of a given event various

other events—e.g. regarding your chance of getting cancer, the class might include various other people like you. But there may be competing such classes that yield different relative frequencies—you may belong both to the class of smokers and the class of those with no family history of cancer. What, then, is the real chance? This is the *reference class problem*.

Some frequentists follow von Mises in requiring the sequences of trials that ground chances to be *infinite*, and thus presumably hypothetical. Then, the ‘chance’ of an outcome-type is identified with its *limiting* relative frequency. (Further *randomness* constraints might also be imposed on the sequences.) Counter-intuitively, such ‘chances’ are then sensitive to the trials’ ordering (a sequence with infinitely many heads and tails can be rearranged to give whatever limiting relative frequency we like). Moreover, the appeal to hypothetical trials, let alone infinitely many of them, may betray the empiricist and scientific scruples that made frequentism appear attractive initially, for such ‘chances’ are not constrained by anything in our experience.

Historically associated with Peirce and Popper, *propensity* accounts of chance postulate primitive dispositions, or tendencies, possessed by various physical systems. Propensity theories fall into two broad categories. According to *single-case* propensity theories, propensities measure a system’s tendencies to produce given outcomes; according to *long-run* propensity theories, propensities are tendencies to produce long-run outcome frequencies over repeated trials. The former have been advocated by the later Popper, Miller, and Fetzer; the latter by the early Popper, Humphreys, and Gillies.

Adopting a long-run view answers a need for testability of propensity attributions, one arguably found wanting for single-case propensity attributions: a long-run attribution may be held falsified if the long-run statistics diverge too much from those expected. However, defining propensities in terms of long-run relative frequencies may render

single-case chance attributions problematic. This poses a dilemma for the long-run propensity theorist: if propensities are linked too closely to long-run frequencies, the view risks collapsing into a variant of frequentism. But if the view is cast so as to make single-case chance attributions possible, it risks collapsing into a variant of the single-case propensity view.

Long-run propensity theories may be motivated by a worry that in a single case there can be factors present that are not part of the description of the chance setup but that affect the chances of various outcomes. If the long-run propensity theorist responds by, in effect, falling back on long-run frequentism, the single-case propensity theorist goes the other way, embracing all causally or physically relevant details as part of the chance setup, determining (though we cannot measure it) the single-case chance for any given trial: each outcome's chance is determined by everything that might influence the setup's evolution. Propensity theories of this type respect some of our physical and causal intuitions, but pay a price epistemically. Since each single-case setup is presumably unique, we cannot use frequencies to estimate the chances or to falsify hypotheses about them.

A final problem, specifically for *conditional* propensities, is *Humphreys' Paradox*. If $\text{Pr}(A|B)$ is a propensity, it seems to have a built-in causal direction, from B to A; the 'inverse' conditional probability $\text{Pr}(B|A)$ can often be calculated, but it appears to get the causal direction wrong. Various authors argue that inverse probabilities cannot be considered propensities, earlier events not having propensities to have arisen from later events; thus, not all conditional probabilities may be interpretable as propensities.

While frequentist and propensity theories have dominated philosophical accounts of chance, a recent recurring proposal is that 'chance' be viewed as a theoretical term similar to others in the sciences, such as 'mass', or 'fitness'. In this post-positivist era,

philosophers mostly agree that such terms cannot be reduced to non-theoretical terms; instead, we may view theoretical terms as implicitly defined by their roles in scientific and philosophical theories. This approach avoids many of the difficulties discussed above, but it may not satisfy philosophers who find something troubling about the very notion of chance (see below). It also renounces giving a philosophical account of chance with *normative* status—e.g. claiming that theorists *should* admit objective chances into QM but not into economics.

Pioneering work by Lewis on the connections between chance and subjective probability—*credence*—has inspired *Humean Best-System* theories. They share these tenets:

(i) chances are defined so that their distinctive connection with credences is rendered transparent (see "Chance and credence" below);

(ii) chances supervene on the entire history of actual events (and not on anything modal that does not itself supervene on the actual);

(iii) chances are determined by the laws of nature—the regularities of a best system (theory) that optimizes the balance of simplicity, strength (covering as many phenomena as possible), and fit (how "typical" actual events are given the chances posited by the system).

Humean Best-System accounts aim to be as acceptable to empiricists as finite frequentism, while avoiding the defects of that account.

Chance and credence

Perhaps the most crucial demand we make of chances is that they guide our bets, expectations, and predictions – that they be “guides to life” in the face of uncertainty.

This role is captured by some chance-credence principle or other, the most common coinage recently being Lewis':

$$\text{Principal Principle (PP)} \quad Cr(A / ch(A) = x \ \& \ E) = x$$

Cr is one's credence function, A is a proposition, $ch(A)$ is the chance of A (presumably time-indexed), and E is further evidence that one may have. In order for PP to be applicable, E cannot be relevant to whether A is true or false, other than by bearing on the chance of A . PP codifies something crucial about chance. A touchstone for any theory of chance is that it should underwrite PP; there is considerable controversy over which theory (if any) can meet this challenge.

Chance and determinism

Determinism is the thesis that any complete past or present state of the world, conjoined with the laws of nature, entails all future events. In a deterministic world, some insist, chance has no work left to do, the entire future being already determined by past events. Philosophers are divided over whether determinism rules out (non-trivial) chances. Since the definition of determinism says nothing about chance, more is needed to argue that determinism rules out chances.

Mellor, Popper, and others who view propensities as fundamental physical loci of indeterminism, see the inference from determinism to nonexistence of chances as immediate. Frequentists such as Venn and Reichenbach see no such inference: intermediate frequencies can exist equally in deterministic and indeterministic worlds. The Humean Best-System approach leaves open whether a deterministic system of laws can include chance laws (although Lewis rejects this possibility). And on the implicit-definition approach, intermediate chances and determinism coexist precisely if our fundamental physical theories are deterministic but some scientific theory postulates

objective probabilities. Statistical mechanics uses chances, but its underpinnings are deterministic, and typical uses of chance in biology and social sciences involve no presumption for or against determinism, as Levi and others have argued.

Nor does indeterminism guarantee the existence of chances. Fundamental physical laws may fail to entail a unique future without being probabilistic. However, if these laws are probabilistic, as some interpretations of QM contend, then chances are apparently guaranteed, on any but a skeptical/subjectivist view.

Skepticism about chance, subjectivism, and exchangeability

Chance is meant to play a certain theoretical role. It is a further matter what, if anything, actually *plays* this role. According to de Finetti, nothing does. “Probability does not exist”, he said, meaning that *chance* does not exist; rather, all probability is subjective. Skepticism about chance is easily assimilated to skepticism about kindred modal notions—possibility, counterfactuals, causation, laws of nature—which seem not to be straightforwardly reducible to non-modal notions, in particular those congenial to an empiricist. And skepticism specifically about chance can be based on further arguments—for one can be skeptical not just about its modality, but also about its putative *degrees*. Subjectivists have also argued that chance is redundant, its alleged role completely discharged by credences. Jeffrey, van Fraassen, Skyrms, and others have developed subjectivist positions in the spirit of de Finetti.

Moreover, the mathematics of chance (unlike the other modal notions) permits a particular eliminativist gloss. A sequence of trials is said to be *exchangeable* with respect to a probability function if the probabilities of trial outcomes are invariant under finite permutations of trials; probabilities may be sensitive to the numbers of outcomes of each kind, but not to their ordering. De Finetti showed that when this condition is met, there is

a unique representation of the probability distribution over the trials as an expectation of simpler probability distributions according to which the trials are independent and identically distributed. For example, if your credences over the results of repeated coin-tossing are exchangeable, then it is *as if* you treat the trials as tosses of a coin of unknown bias, with credences over the possible biases. Subjectivists have argued that this delivers some of the supposed benefits of chance, without any questionable metaphysics.

Conclusion

Many of the perplexities about chance are familiar from other modal notions – its controversial metaphysics, its seeming resistance to reduction, its epistemological recalcitrance, etc. But it has been handled in mathematics and philosophy with rather more precision than those other notions. In the process, still further perplexities have been born. For the foreseeable future, at least in the writings of philosophers and philosophically-minded scientists, chance is probably here to stay.